2103000204023001
EXAMINATION FEBRUARY-MARCH 2024
BACHELOR OF SCIENCE (FOURTH SEMESTER)
MATHEMATICS PAPER-VIII
(MTH-401-MATHEMATICS-VIII)

[Time: As Per Schedule] **[Max. Marks: 50]**

Instructions:
1. Fill up strictly the following details on your answer book
   a. Name of the Examination : **BACHELOR OF SCIENCE (FOURTH SEMESTER)**
   b. Name of the Subject : **MATHEMATICS PAPER-VIII (MTH-401-MATHEMATICS-VIII)**
   c. Subject Code No : **2103000204023001**
2. Sketch neat and labelled diagram wherever necessary.
3. Figures to the right indicate full marks of the question.
4. All questions are compulsory.
5. Follow usual notations and conventions.

Seat No:

Student's Signature

Q.1 Answer any FIVE from the following questions. **10**

1) Show that: $\beta\frac{(m,n)}{m+n}\ \beta\frac{(m+1,n)}{m} = \beta\frac{(m,n+1)}{n}$.
2) Evaluate: $\int_0^\infty x^4 e^{-x} dx$.
3) Evaluate: $\int_1^8 \int_{\frac{y}{2}}^4 x^2\, y^2 dydx$.
4) Describe the region of integration: $\int_0^2 \int_0^4 (x^2 + y^2) dydx$.
5) Find the Laplace transform of $F(t)=e^{at}$.
6) Find $L\{\sinh at\}$.
7) Evaluate: $L^{-1}\left\{\frac{2p+1}{p(p+1)}\right\}$.
8) Evaluate: $L^{-1}\left\{\frac{3}{9p^2-16}\right\}$.

Q.2 Attempt any TWO. **10**

(1) Prove that $\beta(l,m) = \frac{\lceil l \lceil m}{\lceil l+m}$ where $l \succ 0, m \succ 0$.

(2) Show that $\int_0^2 x(8-x^3)^{\frac{1}{3}}\,dx = \frac{16\pi}{9\sqrt{3}}$.

(3) Evaluate: $\int_0^2 \frac{x^2}{\sqrt{2-x}}\,dx$.

Q.3 Attempt any TWO. **10**

1) Find the area enclosed between two parabolas $y^2 = 2x$ and $x^2 = 2y$ using double integral.
2) Change the order of integration of the double integral

$$\int_0^5 \int_{\frac{12}{5}y}^{\sqrt{169-y^2}} f(x,y)\,dydx.$$

3) Describe the region of integration and evaluate it $\int_0^3 \int_{-y}^{y} (x^2+y^2)dxdy$.

Q.4 Attempt any TWO. **10**

1) State and prove second shifting theorem for Laplace transform.
2) Find the Laplace transform of $e^{-t}(3sin2t - 5cosh\,2t)$.
3) Evaluate: $\int_0^\infty te^{-3t}\,sin\,tdt$.

Q.5 Attempt any TWO. **10**

1) State and prove first shifting theorem for inverse Laplace transform.
2) Show that: $L^{-1}\left\{\frac{1}{(p+1)(p-2)}\right\} = -\left\{\frac{e^{-t}-e^{2t}}{3}\right\}$.
3) Evaluate: $L^{-1}\left\{\frac{3p+2}{4p^2+12p+9}\right\}$.

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